

The Mass of the MACHO-LMC-5 Lens Star

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ABSTRACT

We combine the available astrometric and photometric data for the 1993 microlensing event MACHO-LMC-5 to measure the mass of the lens, $M = 0.097 \pm 0.016 M_{\odot}$. This is the most precise direct mass measurement of a single star other than the Sun. In principle, the measurement error could be reduced as low as 10% by improving the trig parallax measurement using, for example, the *Space Interferometry Mission*. Further improvements might be possible by rereducing the original photometric lightcurve using image subtraction or by obtaining new, higher-precision baseline photometry of the source. We show that the current data strongly limit scenarios in which the lens is a dark (i.e., brown-dwarf) companion to the observed M dwarf rather than being the M dwarf itself. These results set the stage for a confrontation between mass estimates of the M dwarf obtained from spectroscopic and photometric measurements and a mass measurement derived directly from the star's gravitational influence. This would be the first such confrontation for any isolated star other than the Sun.

Subject headings: gravitational lensing — stars: low-mass

1. Introduction

Direct measurements of stellar masses provide an essential foundation for theoretical models of stars. Such measurements must be free of model-dependent assumptions about the star’s internal physics and so can be obtained only from the star’s gravitational effects on external objects. Almost all stars with directly measured masses are components of binaries. However, binary stars may not always evolve as do single stars, and thus direct mass measurements of single stars are of prime importance. The Sun’s mass has been measured using two completely independent methods: first by applying Newton’s generalization of Kepler’s Third Law to the motion its companions (the planets); second from its deflection of light from distant stars (Dyson, Eddington, & Davis 1920; Froeschle, Mignard & Arenou 1997). For all stars other than the Sun, direct mass measurements have, until recently, only been possible with the first method. That is, direct mass measurements have been restricted to components of binary systems with orbital periods of decades or less.

All light-deflection methods rely fundamentally on the equation,

$$\theta_E = \sqrt{\kappa M \pi_{\text{rel}}}, \quad \kappa \equiv \frac{4G}{\text{AU } c^2} \simeq \frac{8.14 \text{ mas}}{M_\odot}, \quad (1)$$

where θ_E is the angular Einstein radius, M is the mass of the lens, and π_{rel} is the lens-source relative parallax. Refsdal (1964) was the first to propose that the masses of stars could be measured from light deflection. His method was purely astrometric. By measuring the relative separation of a nearby lens and a more distant source, one could directly determine the relative parallax π_{rel} , the angular impact parameter β , and the maximal deflection $\Delta\theta$. In the simplest (and typical) case, $\beta \gg \theta_E$,

$$\theta_E^2 = \beta \Delta\theta, \quad (\beta \gg \theta_E). \quad (2)$$

Hence, combining equations (1) and (2) leads to a simple expression for the mass. Refsdal (1964)’s method will be carried out for perhaps a dozen nearby stars using the *Space Interferometry Mission* (SIM, Salim & Gould 2000).

A second method was proposed by Gould (1992) and first carried out by Alcock et al. (1995). Here one uses the accelerated platform of the Earth to measure the microlens parallax,

$$\pi_E = \sqrt{\frac{\pi_{\text{rel}}}{\kappa M}}, \quad (3)$$

and combines this with an independent measurement of θ_E to obtain the mass. The method yields the mass and, simultaneously, the lens-source relative parallax π_{rel} :

$$M = \frac{\theta_E}{\kappa \pi_E}, \quad \pi_{\text{rel}} = \pi_E \theta_E. \quad (4)$$

The major problem here is that the number of events that last long enough for the Earth’s acceleration to significantly affect the lightcurve is quite small, and for only a small minority of these events is it possible to measure θ_E . Indeed, there is only one event with a precisely measured mass using this technique (An et al. 2002), and that event is a binary. There are, in addition, two single-star events to which this method has been applied, although the mass measurements are much cruder. For one, OGLE-2003-BLG-238, the mass is measured to only a factor of a few (Jiang et al. 2004). The other is MACHO-LMC-5, which is the subject of this paper.

In addition, Ghosh et al. (2004) have shown that the mass of OGLE-2003-BLG-145/MOA-2003-BLG-45 could be determined precisely provided that the lens-source relative proper motion is measured astrometrically. One variant of this method, proposed by Delplancke et al. (2001), is to measure π_E using the accelerated platform of the Earth and θ_E from the centroid shift of the source during the event (Miyamoto & Yoshii 1995; Hog et al. 1995; Walker 1995). This method would be especially applicable for black-hole microlensing candidates (Bennett et al. 2002; Mao 2002; Agol et al. 2002), which are expected to have large enough θ_E to allow such measurement from the ground and which are dark (and so ineligible for the relative proper-motion method of Ghosh et al. 2004).

Finally, this approach can be extended to a greater number of shorter-duration events by measuring the microlens parallax π_E by comparing the photometric events as seen from the Earth and a satellite in solar orbit (Refsdal 1966; Gould 1995) and combining this with an astrometric measurement of θ_E using a space interferometer (Boden, Shao & Van Buren 1998; Paczyński 1998). Of order 200 such mass measurements will be made by *SIM* (Gould & Salim 1999), which will itself be in solar orbit.

MACHO-LMC-5 is unique in that it has both a measurement of $\boldsymbol{\pi}_E$ derived from the microlensing lightcurve (Alcock et al. 2001; Gould 2004), to which we refer in this paper as a “photometric” quantity, and a completely independently determined full post-event astrometric solution, including both π_{rel} and the lens-source relative proper motion $\boldsymbol{\mu}_{\text{rel}}$ (Alcock et al. 2001; Drake, Cook & Keller 2004). Here, $\boldsymbol{\pi}_E$ is the vector microlens parallax, whose magnitude is π_E and whose direction is that of the lens-source relative motion.

Thus, MACHO-LMC-5 is of particular interest for several reasons. First, it permits three independent tests on the consistency of the measurements. Second, if the measurements pass these consistency tests, they can be combined to obtain a more accurate estimate of the mass. Third, the very high magnification of the microlensing event $A_{\text{max}} \sim 80$ permits one to place very strong constraints on the lens being a close binary rather than a single star. Fourth, the photometric and astrometric measurements can be combined to test the hypothesis that the resolved star that appears to be the lens in the astrometric images is not in fact the lens

but rather is merely a luminous binary companion to a dark substellar object that generated the microlensing event. Finally, the successful completion of all these tests would allow a direct confrontation between the measured mass of a single star and its mass as predicted from photometric and spectroscopic observations. This would be the first such confrontation for any isolated star other than the Sun.

Here we build on the work of Alcock et al. (2001), Gould (2004), and Drake et al. (2004) to carry out the above-described tests, in so far as it is possible today, and outline how these tests can be further refined in the future. The error in our mass estimate, 17%, is the smallest for any direct mass measurement of a single star other than the Sun, and even approaches the precision of measurements of M-dwarf masses from binaries (Delfosse et al. 2000).

2. Brief History

Alcock et al. (2001) originally measured μ_{rel} by analyzing epoch 1999 *Hubble Space Telescope* (*HST*) Wide Field and Planetary Camera (WFPC2) images obtained by Alcock et al. (2000). To do so, they assumed that the two resolved objects in those images were the lens and the source that had been virtually coincident 6.3 years earlier when the event occurred in 1993. They checked for consistency between the direction of μ_{rel} so obtained and the direction of π_{E} (measured in the heliocentric frame) as determined from a microlens parallax analysis of the microlensing-event lightcurve. They found that these position angles agreed at the 1σ level. By combining the astrometric and lightcurve data, they determined both π_{E} and θ_{E} and so, using equation (4), both the mass and distance of the lens. The best estimate of the lens mass was substantially below the hydrogen-burning limit (albeit with moderately large error bars) and so appeared somewhat inconsistent with the hypothesis that the luminous star seen in the images was actually the lens. The inferred, distance, $D_1 \sim 200$ pc, was also quite close relative to the photometric distance inferred from the *HST* photometry. Alcock et al. (2001) suggested that more precise astrometric measurements with the new Advanced Camera for Surveys (ACS) on *HST* could determine the lens-source relative parallax π_{rel} , and so test the distance estimate and hence effectively test the entire procedure for inferring both the mass and distance of the lens.

In the meantime, Gould (2004), building on the work of Smith et al. (2003), discovered that microlens parallax measurements of relatively short events (with timescales $t_{\text{E}} \lesssim \text{yr}/2\pi$) are subject to a four-fold degeneracy composed of two two-fold ambiguities. While one these ambiguities (the so-called “constant-acceleration” degeneracy) has only a very small effect on the event parameters, the other (the “jerk-parallax” degeneracy) can affect π_{E} by quite a lot. Gould (2004) showed that the alternate jerk-parallax solution for MACHO-LMC-5 had

a somewhat larger mass and a much larger distance, $D_1 \sim 450$ pc.

Most recently, Drake et al. (2004) have carried out the ACS measurements advocated by Alcock et al. (2001), and these have yielded both a more precise measurement of μ_{rel} and a new measurement of π_{rel} . Drake et al. (2004) were able to conduct two consistency checks. First, they found that their trigonometric parallax measurement was consistent with the lens distance derived by Gould (2004) for his alternate ($D_1 \sim 450$ pc) solution. Second, they found that the direction vector $\boldsymbol{\mu}_{\text{rel}}/\mu_{\text{rel}}$ determined from astrometry was consistent with the direction vector $\boldsymbol{\pi}_{\text{E}}/\pi_{\text{E}}$ found by Gould (2004) for this alternate solution.

In fact, the proper motion derived from the WFPC2 observations proved very accurate and agreed to within $0^\circ.2$ with the direction from the new ACS measurements (both in 2000 celestial coordinates). However, Alcock et al. (2001) reported these results only in ecliptic coordinates, and they made a 14° degree error when they translated from celestial to ecliptic. This transcription error significantly affected the mass and distance estimates of both Alcock et al. (2001) and Gould (2004) when they used the direction of proper motion as a constraint in their solutions. The correction of this transformation error, by itself, resolves the most puzzling aspects of the solutions obtained by Alcock et al. (2001) and Gould (2004). If the correct direction had been incorporated into the fits, both solutions would have moved to within 1σ of the hydrogen-burning limit. The Gould (2004) solution would have moved to ~ 530 pc and so would have been in better agreement with the photometric parallax, although the Alcock et al. (2001) solution would have actually moved to an even shorter distance, ~ 160 pc.

2.1. ACS Astrometry

In this paper, we will make extensive use of the astrometric measurements of Drake et al. (2004), sometimes combining them with other real and/or hypothetical data. To do so, we must fit to the original astrometric data. We find that when we fit these data alone, the results differ very slightly (much less than 1σ) from the results reported by Drake et al. (2004). For consistency, we always use the values from our own fits, although the difference has no practical impact on any of the derived results. In fitting the Drake et al. (2004) data, we exclude the 2002 F814 point as they also did (A. Drake 2004, private communication). This point is a significant (3σ) outlier, which contradicts a F606 measurement taken at almost exactly the same time.

Our fit yields the following parameter estimates: relative parallax $\pi_{\text{rel}} = 1.780 \pm 0.185$ mas; relative proper motion $\mu_{\text{rel}} = 21.381 \pm 0.022$ mas yr $^{-1}$; proper motion compo-

nents $\mu_{\text{rel,East}} = 17.547 \pm 0.029 \text{ mas yr}^{-1}$; and $\mu_{\text{rel,North}} = -12.217 \pm 0.022 \text{ mas yr}^{-1}$; and position angle $\phi = 124.85 \pm 0.08$.

As noted by Drake et al. (2004), the residuals for the WFPC2 point are quite small compared to their reported errors, with $\Delta\chi^2 = 0.11$ for 2 degrees of freedom (dof). This may imply that the errors were overestimated by Alcock et al. (2001). However, since there is a 10% probability of having such low residuals by chance, no definite conclusions can be drawn regarding a possible overestimation of the error bars.

3. Sources and Consistency of Data

In this paper, we will draw together four sources of data to measure the mass of MACHO-LMC-5. We first summarize these sources, then discuss a series of tests that we have carried out to determine whether they are consistent with each other. Only after these tests are successfully concluded do we combine the data.

3.1. Data Sources

The primary data set is the original MACHO SoDoPHOT pipeline photometry of the event, which occurred in 1993. These data have already been analyzed by Alcock et al. (2001) and Gould (2004). They consist of 352 points in the non-standard MACHO red filter (hereafter R_M) and 265 points in the non-standard MACHO blue filter (hereafter V_M). We slightly deviate from previous authors by recursively removing outliers and renormalizing the errors so as to enforce χ^2 per degree of freedom (dof) equal to unity in each bandpass separately. We repeat this procedure until all 3.5σ outliers are removed. This removes three R_M points and one V_M point, all greater than 3.9σ . The next largest deviation is at 3σ , but with more than 600 points, such a deviation is consistent with Gaussian statistics and so cannot be considered an outlier. The error renormalization factors are 0.79 in R_M and 0.81 in V_M .

Gould (2004) had argued against blindly applying this renormalization procedure because the mass determination is dominated by the relatively small number of points during the event, while χ^2/dof is dominated by the much larger number of baseline points taken over several years. However, we find that χ^2/dof is similar for both these subsets and therefore proceed with the renormalization described above. Of the four eliminated points, only one is during the event, the R_M point at JD = 2490015.14. From Figure 1 of Gould (2004), it can be seen that this point is a clear outlier with an abnormally large error bar.

For isolated faint stars, SoDoPHOT (“Son of DoPHOT”) reports only photon noise errors, which of course cannot be overestimates. However, in crowded fields, SoDoPHOT follows DoPHOT (Schechter, Mateo & Saha 1993) in “padding” the photon noise as it subtracts out surrounding stars. This additional padding is correct in some average sense, but may be an overestimate or underestimate in individual cases. Hence, for events of particular interest, it is worthwhile to investigate these error bars more closely.

For Gaussian errors, removal of outliers greater than $\sigma_{\max} = 3.5$ artificially reduces χ^2 by $\sim (2/\pi)^{1/2} \sigma_{\max} \exp(-\sigma_{\max}^2/2) \sim 0.6\%$ and so understates the size of the error bars by 0.3%. This difference has no practical effect on the results reported here.

The second data source is the *astrometric* measurement of the lens-source separation made by analyzing epoch 1999 *Hubble Space Telescope* (*HST*) Wide Field and Planetary Camera (WFPC2) images originally obtained by Alcock et al. (2000). Alcock et al. (2001) and Gould (2004) have previously combined the proper motion measurement, μ_{rel} , derived from these data with the above-mentioned lightcurve data to estimate the mass and distance of the lens.

The third data source is the *photometric* measurements of the source brightness made from the same *HST* WFPC2 images. These data help constrain the lightcurve fit and thus tighten the errors on the mass and distance measurements. Alcock et al. (2001) made use of these measurements in their constrained fit, but Gould (2004) did not.

The final data source is the new astrometric measurements made by Drake et al. (2004) using ACS. These have yielded both an improved measurement of μ_{rel} and a new parallax measurement, π_{rel} . Both the WFPC2 and ACS measurements are listed in Table 1 of Drake et al. (2004).

3.2. Consistency Checks

3.2.1. Source Color and Magnitude

We first wish to combine the original SoDoPHOT photometry of the event with the flux measurement of the source made by Alcock et al. (2001) after the event was over and the source was well separated from the lens. The superb resolution of *HST* virtually ensures that all blended light is removed from the source with the possible exception of a wide-binary companion to the source, which we discuss below. This *HST* source photometry can be compared with the source flux that is returned as a parameter by the microlensing fit. To do so, one must first translate the *HST* photometry into the MACHO bands. This

can be done by directly comparing the flux levels recorded by *HST* and SoDoPHOT for an ensemble of other stars in the field. While each of these is blended in the MACHO images, the blending is equally likely to contaminate the object or the “sky” determination. So it should introduce scatter but not a systematic bias. This scatter is smaller for brighter stars, but unfortunately the PC field is not big enough to contain many bright stars. Based on a comparison of 18 relatively bright stars (and constraining the fits by the two color-color slopes, $V - V_M \propto -0.20(V_M - R_M)$, $R - R_M \propto +0.18(V_M - R_M)$ reported by Alcock et al. 1999), we find that the *HST* data imply

$$f_{s,V} = 23.27 \pm 1.34, \quad f_{s,R} = 23.84 \pm 1.23, \quad \frac{f_{s,R}}{f_{s,V}} = 1.024 \pm 0.016 \quad (HST), \quad (5)$$

where the errors and covariances are derived by enforcing χ^2/dof in the fit. The error in the color calibration is substantially smaller than the errors in the flux calibrations because the latter are highly correlated, with correlation coefficient $\rho = 0.962$. These results may be compared to the source flux levels derived from the fit to the SoDoPHOT data alone,

$$f_{s,V} = 28.97 \pm 3.99, \quad f_{s,R} = 29.50 \pm 4.16, \quad \frac{f_{s,R}}{f_{s,V}} = 1.018 \pm 0.018 \quad (\text{SoDoPHOT lightcurve}). \quad (6)$$

The two determinations differ by $\sim 1.3\sigma$ in each of the two (highly correlated) bands separately and by $\sim 0.3\sigma$ in the color. Since the two photometric measurements are consistent, they can be combined. We then find that the χ^2 minimum increases by 2.2 for two additional dof, confirming the consistency of the two pairs of measurements.

The one possible caveat is that the microlensing fit gives the flux of the source that was magnified during the event while the *HST* measurement gives all the flux from stars within about 100 mas of the source center, corresponding to about 5000 AU at the distance of the Large Magellanic Cloud (LMC) (Alves 2004). Significant sources of light that lay beyond 50 mas would have shown up in subsequent ACS images. Now, as we will show below, the angular Einstein radius is $\theta_E \sim 1$ mas. Any significant light source within about $(1/3)\theta_E$ of the source would have betrayed itself during the event. This still leaves the possibility that the source has a binary companion between 17 and 2500 AU. While we cannot rule out such a possibility, there are several lines of argument against it. First, the discrepancy (eqs. [5] and [6]) between the two measurements (though not statistically significant) is of the *wrong sign* to be accounted for by a binary companion. Second, the good agreement in the colors shows that any companion must be either of nearly the same color as the source or quite faint. If the former, the two stars should also be of roughly the same magnitude, in which case one would expect the discrepancy to be much larger than observed (and, again, in the opposite direction). Third, to produce $\sim 10\%$ or more of the light, the companion mass would have to be $> 70\%$ of the primary. If LMC binaries are similar to those studied

by Duquennoy & Mayor (1991) in the Galactic disk, the fraction of stars with companions in the required mass and separation ranges is only $\sim 7\%$. Hence, it is unlikely, though not impossible, that such a companion exists and is significantly corrupting the measurement.

3.2.2. Direction of Motion

We therefore begin by incorporating the WFPC2 photometric measurements into the lightcurve fit, taking account of both their errors and covariances. The resulting contour plot for the vector parallax $\boldsymbol{\pi}_E$ (in the geocentric frame) is shown in Figure 1. This figure should be compared to Figure 1 from Gould (2004). Each of the two minima is consistent between the two figures at the 1σ level. This is to be expected, since the additional photometric data are consistent with those used by Gould (2004). However, the errors are substantially smaller, both because the photometric errors have been renormalized by a factor ~ 0.8 and because of the additional higher-precision *HST* baseline photometry of the source.

We can now ask whether the direction of motion (in the heliocentric frame) implied by each of these lightcurve (ϕ_{lc}) solutions is consistent with the direction of proper motion (ϕ_{ast}) that we derive from the *HST* ACS and WFPC2 data of Drake et al. (2004). For the southeast solution, which the Drake et al. (2004) parallax measurement demonstrates to be the correct one, the comparison yields

$$\phi_{ast} = 124^\circ.85 \pm 0^\circ.08 \quad \phi_{lc} = 132^\circ.3^{+3^\circ.4}_{-5^\circ.3}. \quad (7)$$

Hence, the photometrically and astrometrically determined directions are consistent at about the 1.3σ level. The other (northwest) solution has a direction $\phi_{lc} = 136^\circ.5^{+2^\circ.2}_{-2^\circ.5}$. However, because the χ^2 surface deviates strongly from a parabola, the discrepancy with the proper-motion data is only at about the 2.8σ level.

3.2.3. Parallax

Since the astrometrically determined direction of motion is consistent with the value derived from the lightcurve (at least for the southeast solution), we combine the astrometric and photometric data. As explained by Alcock et al. (2001) and Gould (2004), this permits a full solution for the event, including the mass M of the lens and lens-source relative parallax π_{rel} . We plot the result in the $[\log M, (M - m)_0]$ plane, where $(M - m)_0$ is the *lens* distance modulus, i.e., corresponding to $\pi_l = \pi_{rel} + \pi_s$ and where we have adopted $\pi_s = 20 \mu\text{as}$ for the source, which resides in the LMC. Figure 2 shows the resulting likelihood contours. Note

that each set of contours is offset from its respective minimum. The minimum of the short-distance (northwest) solution is actually higher by $\Delta\chi^2 = 6$, which is a reflection of the mild direction discrepancy found for this solution in § 3.2.2.

Also shown in Figure 2 is the best fit and 1σ error bar for the parallax determination based on the astrometric data of Drake et al. (2004), $\pi_{\text{rel}} = 1.780 \pm 0.185$ mas. This corresponds to a distance modulus

$$(M - m)_0 = 8.72^{+0.24}_{-0.21} \quad (\text{trig parallax}). \quad (8)$$

To make an algebraic comparison, we fit the $\Delta\chi^2 < 1$ region of the rightward contours to a parabola and find,

$$\log M = -1.023 \pm 0.084, \quad (M - m)_0 = 8.683 \pm 0.144, \quad \rho = 0.921, \quad (\text{lc} + \boldsymbol{\mu}_{\text{rel}}) \quad (9)$$

where ρ is the correlation coefficient. Hence, the lens distance derived from the lightcurve/proper-motion analysis is consistent with the trig parallax at the 1σ level.

4. Mass, Distance, and Velocity of Lens

Since the two measurements are consistent, we combine them. The results are shown in Figure 3 and can be represented algebraically by,

$$\log M = -1.013 \pm 0.073, \quad (M - m)_0 = 8.702 \pm 0.124, \quad \rho = 0.896. \quad (\text{lc} + \boldsymbol{\mu}_{\text{rel}} + \pi_{\text{rel}}) \quad (10)$$

These figures correspond to a best-fit mass and distance

$$M = 0.097 \pm 0.016 M_{\odot}, \quad D_l = 550 \pm 30 \text{ pc}. \quad (11)$$

This best fit has $\chi^2 = 605.39$ compared to $\chi^2 = 601.68$ for the lightcurve alone. That is, $\Delta\chi^2 = 3.71$ for 5 additional dof.

Also shown in Figures 2 and 3 are the estimates of the mass and distance of the lens as derived by Alcock et al. (2001) from photometric *HST* data. These are consistent with the microlensing measurement at the 1σ level.

The (U, V, W) velocities of the lens toward the Galactic center, the direction of Galactic rotation, and the north Galactic pole are

$$U = 43.6 \pm 1.9 \text{ km s}^{-1}, \quad V = -60.8 \pm 8.3 \text{ km s}^{-1}, \quad W = 26.6 \pm 5.7 \text{ km s}^{-1}, \quad (12)$$

with correlation coefficients $\rho_{UV} = -0.90$, $\rho_{UW} = -0.74$, $\rho_{VW} = 0.93$, where we have taken account of the source motion (van der Marel et al. 2002) and the motion of the Sun relative

to the local standard of rest (Dehnen & Binney 1998). The uncertainties are dominated by the error in the radial-velocity measurement, $v_r = 49 \pm 10 \text{ km s}^{-1}$ (Alcock et al. 2001), and this fact accounts for the high correlation coefficients.

Finally, for reference we note that, from equations (1) and (3), these determinations of the mass and distance imply

$$\theta_E = 1.19 \pm 0.07 \text{ mas}, \quad \pi_E = 1.51 \pm 0.17, \quad (13)$$

where we have taken account of the correlations in determining the errors.

5. Constraints on Binarity

An important application of microlensing mass measurements is the opportunity they afford to confront theoretical models that attempt to predict the masses of stars from their spectroscopic and photometric properties. Crucial to such a comparison is the determination that the “star” is in fact a single object and not a close stellar binary or a binary composed of a star and a brown dwarf. In the former case, the photometric properties would be a composite and in both cases the microlens mass would not be the mass of the star dominating the light. It is equally crucial that the luminous star whose visible properties are being measured is actually the lens whose mass was measured during the microlensing event, as opposed to a luminous companion of a dark object (e.g. a brown dwarf) that generated the microlensing event. In this section we investigate how well both of these concerns can be addressed with current and/or future data.

5.1. Limits on Close Binaries

A close binary approximates a Chang & Refsdal (1979, 1984) lens with sheer $\gamma = [d/(q^{1/2} + q^{-1/2})]^2$, where $d\theta_E$ is the angular separation of the two components and q is their mass ratio (Dominik 1999; Albrow et al. 2002). These have caustics of full angular width $4\gamma\theta_E$, which, if traversed, would give rise to obvious deviations from a point-lens lightcurve. For a continuously sampled lightcurve, the best way to avoid the caustic is for the source to pass at an angle of 45° relative to the binary axis. In this case, the caustic is just barely nicked if $u_0 = 2^{1/2}\gamma$, where u is the lens-source separation in units of θ_E and u_0 is its minimum value (i.e., impact parameter). For MACHO-LMC-5, the sampling is far from uniform, and the tightest simple constraint is obtained from the highest point, which comes almost exactly one day after the peak and therefore is at $u = u_{\text{high}} = 0.029$. The largest circle that one can

inscribe in the central caustic has radius $u \sim \gamma$. From this we derive,

$$\gamma = \frac{d^2}{(q^{1/2} + q^{-1/2})^2} < u_{\text{high}} = 0.029, \quad (14)$$

since otherwise this point would have landed in the caustic and so would have been much more magnified than it actually is. Since the lens is much closer than the source, the Einstein radius is essentially equal to the projected Einstein radius, $r_E = \tilde{r}_E = \text{AU}/\pi_E = 0.66 \text{ AU}$. Hence the above limit can be expressed in terms of the projected separation between the binary components, $r_\perp \equiv dr_E$,

$$r_\perp < 0.11(q^{1/2} + q^{-1/2})^2 \text{ AU}. \quad (15)$$

Unless we are very unfortunate to see a widely separated pair projected along the line of sight, or unless the companion is of such low mass as to be uninteresting, the putative companion would cause the source to move by 10 km s^{-1} or more. This would in principle be detectable by spectroscopic measurements.

While close binaries deviate most sharply from point lenses inside their caustics, they do show significant deviations in the surrounding regions as well (see Fig. 1 from Gaudi & Gould 1997). A detailed accounting of these deviations would strengthen the limit in equation (15) but, given the sparse sampling of MACHO-LMC-5, the improvement would most likely be modest.

5.2. Limits on Wide Binaries

A similar argument places a limit on wide companions, which also give rise to Chang-Refsdal caustics. In this case, $\gamma = qd^{-2}$, so

$$r_\perp > 3.9 q^{1/2} \text{ AU}, \quad (16)$$

corresponding to $7.2 q^{1/2} \text{ mas}$.

5.3. Constraints on the Dark Lens Hypothesis

As shown in Figures 2 and 3, the best-fit mass lies close to the hydrogen burning limit. It is therefore possible in principle that the lens is not actually the red star seen in the *HST* images but rather an invisible brown-dwarf companion to it. To what extent can this scenario be constrained by the available data?

As pointed out by Drake et al. (2004), even if one relaxes the assumption that the red star and the microlensed source were coincident at the time of the event, the remaining astrometric measurements “point back” to a relative offset $\Delta\theta$ very close to zero at the peak of the event. Specifically, we find,

$$\Delta\theta_N = -1.2 \pm 7.7 \text{ mas} \quad \Delta\theta_E = -2.9 \pm 7.5 \text{ mas.} \quad (17)$$

As discussed by Drake et al. (2004), the smallness of these values relative to the errors most likely reflects an overestimation of the *HST* WFPC2 errors by Alcock et al. (2001). However, to be conservative, we ignore this possibility. Somewhat stronger constraints can be obtained by noting that in this relaxed solution, the parallax error grows from 0.185 mas to 0.291 mas, and that this error is fairly strongly correlated with $\Delta\theta$. However, the microlensing analysis independently constrains the parallax to be $\pi_{\text{rel}} = 1.81 \pm 0.12 \text{ mas}$, and this constraint can be added into the fit. We then find,

$$\Delta\theta_N = -0.8 \pm 5.6 \text{ mas} \quad \Delta\theta_E = -2.5 \pm 6.8 \text{ mas.} \quad (18)$$

These results indicate that, at the 2σ level, the M dwarf must have been within about 13 mas of the source at the time of the event. On the other hand, from the argument in § 5.2, the M dwarf could not have been too close to the source if it were not actually the lens. By hypothesis, the putative brown-dwarf lens is below the hydrogen-burning limit while the M dwarf is above it, so $q > 1$. Hence,

$$\Delta\theta = \frac{r_{\perp}}{D_1} > 7.2 \text{ mas.} \quad (19)$$

Equations (18) and (19) leave only a fairly narrow range of allowed separations. For face-on circular orbits and for a total binary mass $M_{\text{tot}} = 0.2 M_{\odot}$, these limits correspond to a period range

$$18 \text{ yrs} < P < 44 \text{ yrs} \quad (\text{allowed periods}). \quad (20)$$

Even assuming a mass ratio $q = 2$, the amplitude of the M dwarf motion would be between 2.4 mas and 4.3 mas. These amplitudes are quite large relative to the $\sim 0.3 \text{ mas}$ errors achieved for single epoch *HST* ACS images. Hence, in principle, this scenario could be much more tightly constrained by future observations.

6. Future Confrontations

From Figure 3, the photometrically derived mass and distance of MACHO-LMC-5 are consistent with the values of these properties derived by combining the astrometric and

microlensing data. The error bars for both determinations could be improved significantly by obtaining additional data and by improving the analysis.

On the photometric side, both the mass and the luminosity of the M dwarf are inferred from its color. Apart from the error in measuring this quantity, these inferences suffer from the intrinsic dispersions of mass and luminosity at fixed color. A substantial part of this dispersion is due to metallicity. Drake et al. (2004) have argued that the kinematic data are consistent with either a disk or a thick-disk star, and therefore a range of metallicities of about 1 dex. Hence, spectroscopic determination of the M dwarf’s metallicity would go a long way toward shrinking the photometry-based mass/distance error bars.

On the astrometric/microlensing side, there are three paths to improvement. First, of course, the distance determination could be improved by a better trigonometric parallax measurement. Moreover, because the mass and distance measurements are highly correlated (see eq. [10]), a more accurate distance would also improve the mass determination. Unfortunately, significantly better parallax measurements will not come cheaply. From a comparison of equations (8) and (9), the $185\,\mu\text{as}$ error from the ACS astrometry is about 50% larger than the distance estimate achieved from microlensing (and the proper-motion measurement) alone. A plausible target for a significant improvement would be a $100\,\mu\text{as}$, or better yet $50\,\mu\text{as}$ measurement. These would yield mass determinations with fractional precisions of 13% and 10%, respectively.

Note that even if the distance were known exactly, the microlens mass measurement error could only be reduced to 7.5%.

If it were only necessary to consider the statistical errors, such improvements could be achieved by multiplying the total length of ACS observations by 3 and 14 respectively. However, systematic errors may become important, and the discrepancy between the F606 and F814 measurements implies that caution is warranted.

A parallax measurement by *SIM* might also prove feasible. This seems impossible at first sight because the M dwarf has $V = 22.7$ whereas the magnitude limit of *SIM* is often said to be $V < 20$. However, what fundamentally limits *SIM* at faint magnitudes is the number of sky photons that enter its $1''$ radius stop. If we ignore this sky noise for the moment, a $50\,\mu\text{as}$ measurement at $V = 22.7$ would require an observation of only about 1 hour. An additional 30-minute observation of the $V = 21$ source would yield a $30\,\mu\text{as}$ measurement, for a combined error in the relative offset of $60\,\mu\text{as}$. As in the case of the ACS measurements, only two epochs (in two orientations) would be required because it is known from the microlensing event that the two stars were virtually coincident in 1993. We find that a total of 4 pairs of observations (each pair totaling 90 minutes) would yield a relative

parallax error of $42 \mu\text{as}$. The sky is $V \sim 22.7 \text{ mag arcsec}^{-2}$ (so $V \sim 21.5$ inside the *SIM* stop), which would mean that the observation time for the lens (but not the source) would have to be roughly tripled relative to the naive estimates. However, *SIM* has a very broad bandpass and the M dwarf is very red, which may imply that much of the astrometric signal will come in well above the sky. Hence, the required duration of the exposures cannot be properly estimated until the details of *SIM*'s throughput are worked out in greater detail. In any event, it appears that *SIM* could achieve a substantial improvement in the parallax measurement without prohibitive observing time.

The second potential path would be to obtain better photometry of the source. Recall that the *HST* flux measurement error was dominated by the problem of aligning the WFPC2 and SoDoPHOT photometry, which was exacerbated by the small number and faint flux levels of stars in the small PC chip. Since the source and lens are now well separated, one could image them using the much larger ACS camera and so align the photometry using a large number of relatively bright stars. If the photometric error were reduced from the present 5.5% to 2%, then the mass error would be reduced from 17% to 15%. If this were combined with a $50 \mu\text{as}$ parallax measurement, the mass error would be reduced from the above-mentioned 10% to 7%, while perfect knowledge of the distance would, under these circumstances, reduce the mass error to 4%.

The third path would be improved microlensing data reductions. All the microlensing analysis has been conducted on the basis of the original MACHO SoDoPHOT pipeline photometry. This pipeline produced 10^{10} measurements of very high quality, but with modern image-subtraction routines, it may be possible to do better. However, since the SoDoPHOT errors have been renormalized (see § 3.1) some of this improvement has already been achieved.

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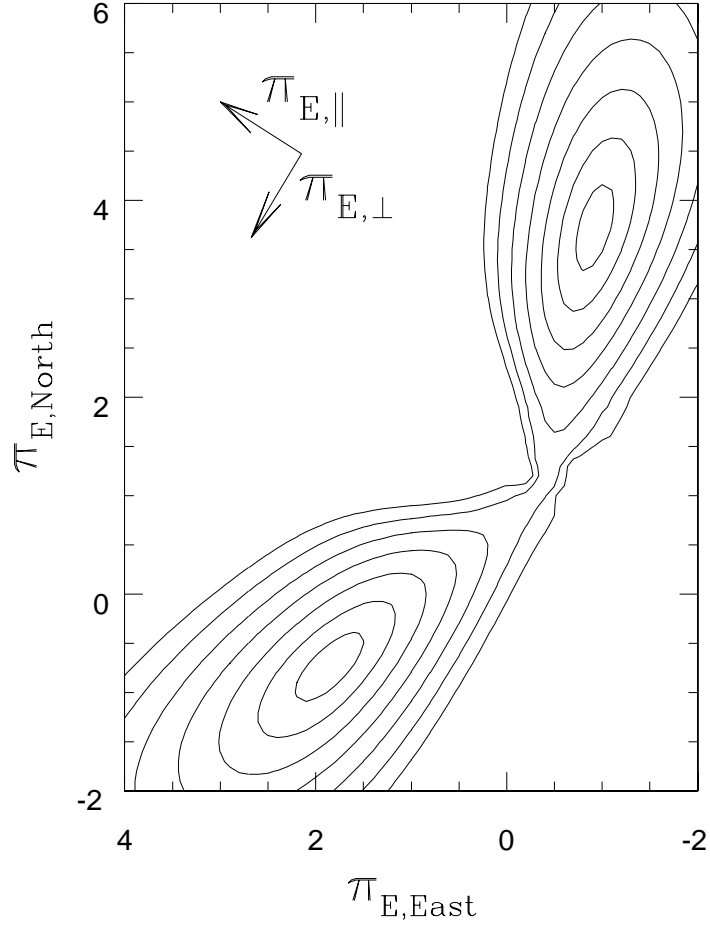


Fig. 1.— Likelihood contours in the π_E plane shown at $\Delta\chi^2 = 1, 4, 9, 16, 25, 36$, and 49 relative to the minimum. This should be compared to Figure 3 of Gould (2004). The errors here are smaller, partly because of error renormalization and partly because of the addition of *HST* baseline photometry of the source from Alcock et al. (2001).

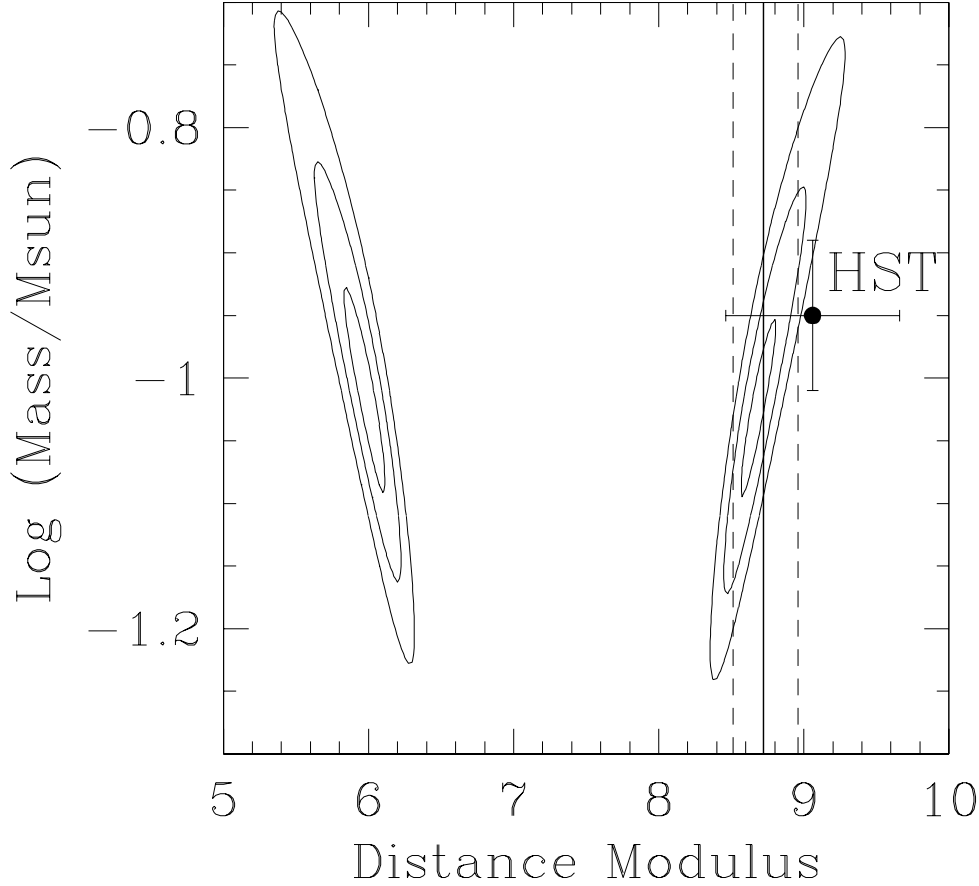


Fig. 2.— Likelihood contours ($\Delta\chi^2 = 1, 4, 9$) for the log mass M and distance modulus $(M - m)_0$ of MACHO-LMC-5 based on the lightcurve of the event, the source-flux measurement of Alcock et al. (2001), and constrained by the proper-motion measurement of Drake et al. (2004). Each set of contours is shown relative to its own minimum. The left-hand minimum is actually higher than the one at the right by $\Delta\chi^2 = 6$. The vertical lines show the best-fit distance modulus and 1σ confidence interval derived from the trig parallax measurements of Drake et al. (2004), while the point with error bars shows $\log M$ and $(M - m)_0$ as determined photometrically from *HST* WFPC2 data by Alcock et al. (2001). Note that the photometric (contours) and astrometric (vertical lines) determinations of $(M - m)_0$ are in agreement at better than 1σ .

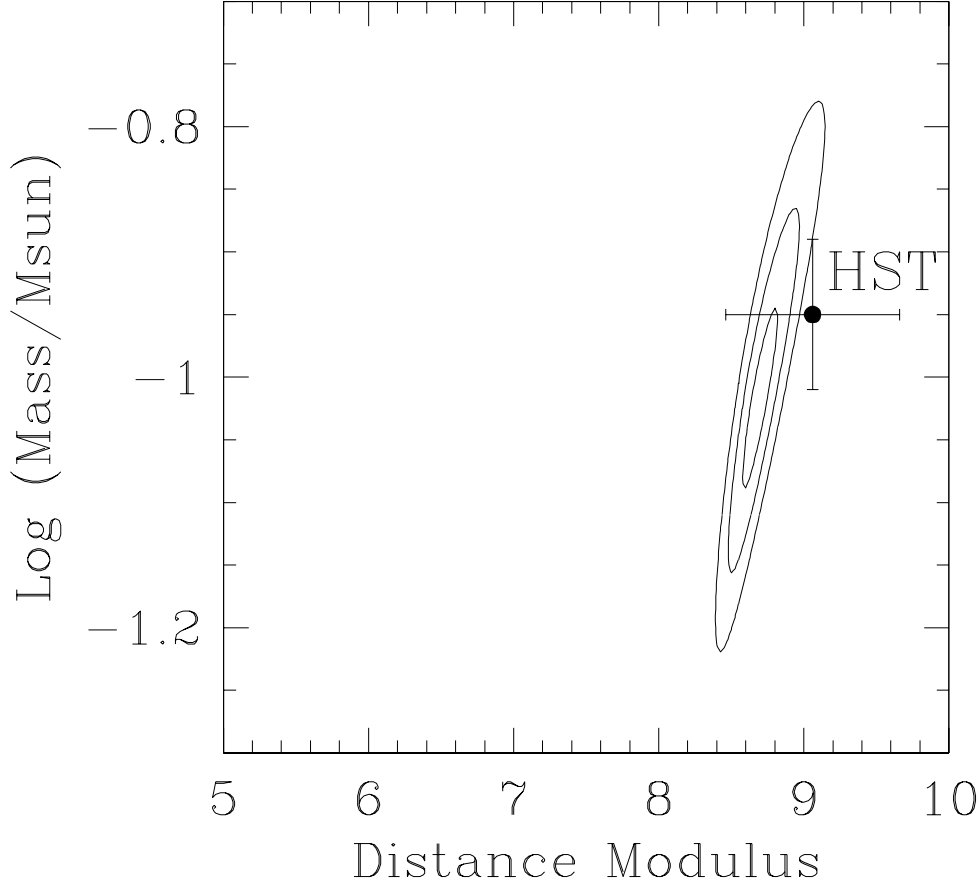


Fig. 3.— Similar to Fig. 2 except that the microlensing/proper-motion determination has now been combined with the lens-source relative parallax measurement of Drake et al. (2004). The errors are 17% in the mass and 6% in the distance. There is good agreement with the mass and distance estimates based on *HST* photometry, shown as a point with error bars.